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ABSTRACT

This instructional unit uses an intuitive approach in introducing the concept of congruent transformations. Extensive use is made of worksheets and manipulative methods. In the latter stages, the SSS, ASA, and SAS theorems are presented. The unit concludes with geometric proofs requiring the use of the fact that corresponding parts of congruent triangles are congruent. (LS)

ED 070646

CONGRUENT TRANSFORMATIONS

A WORKSHOP APPROACH FOR GRADE 9 STUDENTS

SE 015 357

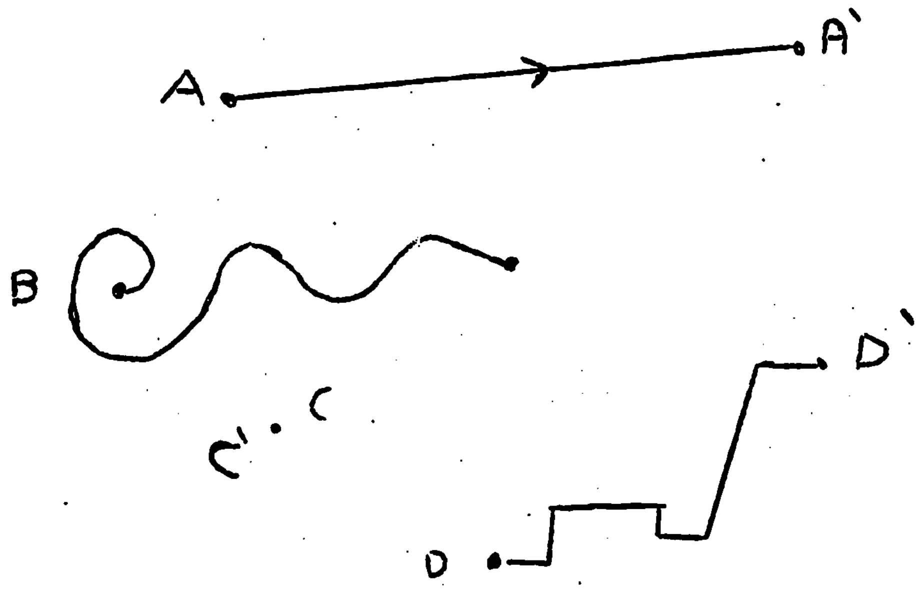
CONGRUENT TRANSFORMATIONS

What is a Transformation?

Another word that could be used instead of transformation is ....
....

A TRANSFORMATION IS A MOVEMENT OF POINTS (change in position).

On a given signal four boys A, B, C and D moved to new positions in the classroom, following the paths shown.



The new position of A is called A', <u>the image</u> of <u>A</u> pronounced A-dash or A-prime.

Mark B'.

What did C do? ....

C is called an <u>invariant</u> point.

D' is the .... of ....

****

REFLECTION:

1.) A mirror can cause the transformation of points.

(cont'd on page 2)

CONGRUENT TRANSFORMATIONS

REFLECTION:

Place your mirror so that you can stand and look at your image.

What happens to your image as you:

1.) walk towards the mirror 1.) ....
2.) back away from the mirror 2.) ....
3.) wink your right eye 3.) ....
4.) touch your left ear 4.) ....
5.) wave your right hand 5.) ....
6.) turn on the spot clockwise 6.) ....
7.) side step to your right 7.) ....

The reflection operation is a "sense-reversing" operation.

8.) Which is taller, you, or your image 8.) ....
9.) Which has the bigger face 9.) ....
10.) If you stand 1 foot in front of the mirror, where does your image appear to be? 10.) ....
11.) If you touch the mirror with a finger, what does the image do? 11.) ....

What is the name given to this point? ....

12.) If you are 3 feet from the mirror, how far are you from your image? 12.) ....

2.) CLASS EXERCISE:

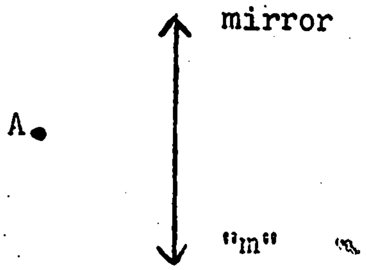
You need two students who look alike and an imaginary mirror. One student acts as the image of the other.

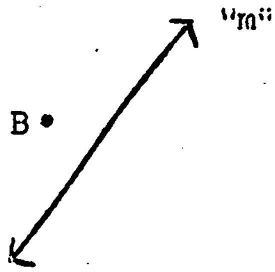
Question: What basic "rule" must be obeyed by the image at all times?

Answer: ....

....

....





Sketch A' and B'

What can you say about the line "m" and the line segment $\overline{AA'}$

....

....

Similarly "m" is the <u>mediator</u> (perpendicular bisector) of $\overline{BB'}$

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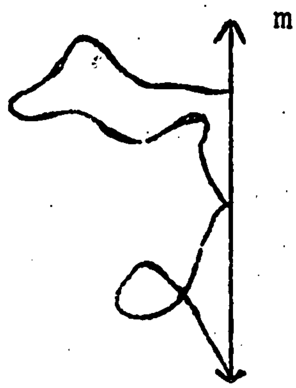
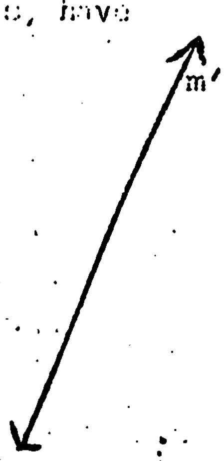
3.) Draw the image of this figure. (free hand)



4.) As you sketch a figure on one side of these mirror lines, have your neighbour draw the image.



4.) What do you notice about:

a.) The distance of your pencils from the mirror line at any instantaneous moment? ....

b.) The imaginary line between your pen point and your partner's pen point at any instantaneous moment? ....

c.) The final sketches you obtain? ....

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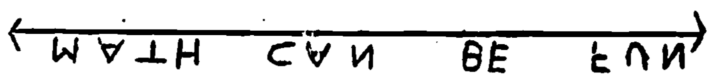
5.) Place your mirror on the line marked "m"



What words do you read? ....
Draw the reflections of the following words (check with your mirror)

a.) REFLECTION b.) SKRIMPZIN c.) STAGE

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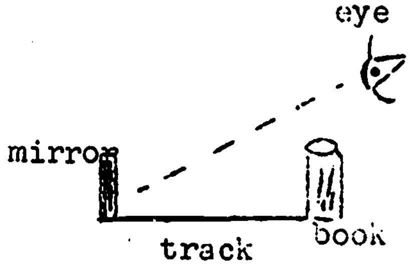
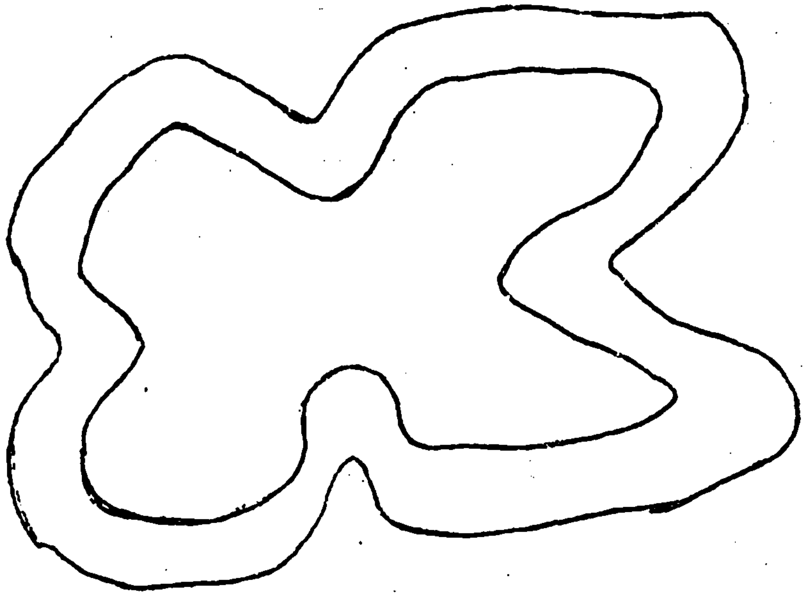
6.) Arrange your mirror and a book so that you can only see this race track by looking in the mirror.

With a pencil, try to move around the track, keeping within the fences. Have your neighbour time you. Record the number of "crashes".





Time ....

Number of crashes ....

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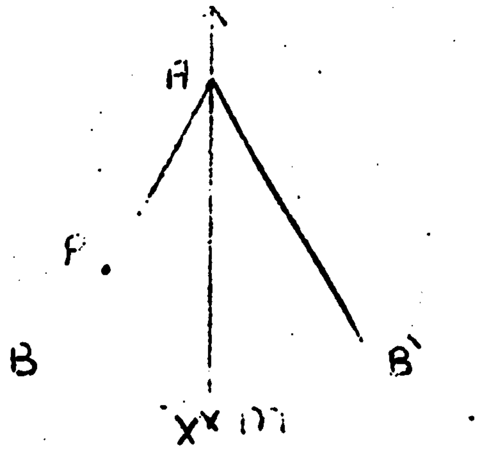
7.)



B` is the image of B after reflection in "m"

What can you say about:

i.) line segments $\overline{AB}$ and $\overline{AB'}$ i.) ....

ii.) line segment $\overline{BB'}$ ii.) ....

(cont'd on page 6)

7.) (cont'd from page 6)

What can you say about:

iii.) angles $\angle BAX$ and $\angle B'AX$ iii.) ....

iv.) $\triangle BAB'$ iv.) ....

P is a point on AB .

By using a ruler (and <u>only</u> for the purpose of drawing straight lines) locate P' .

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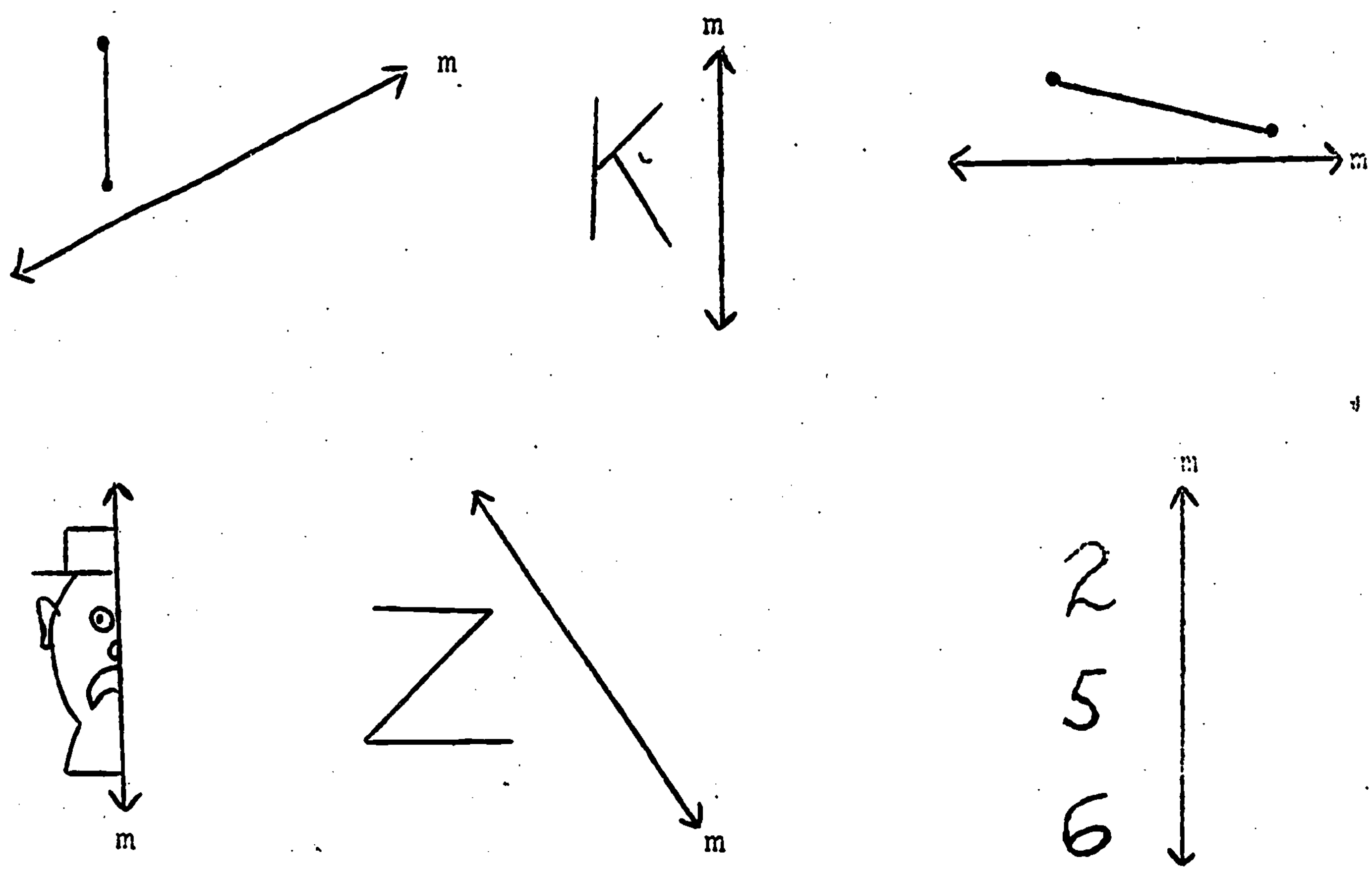
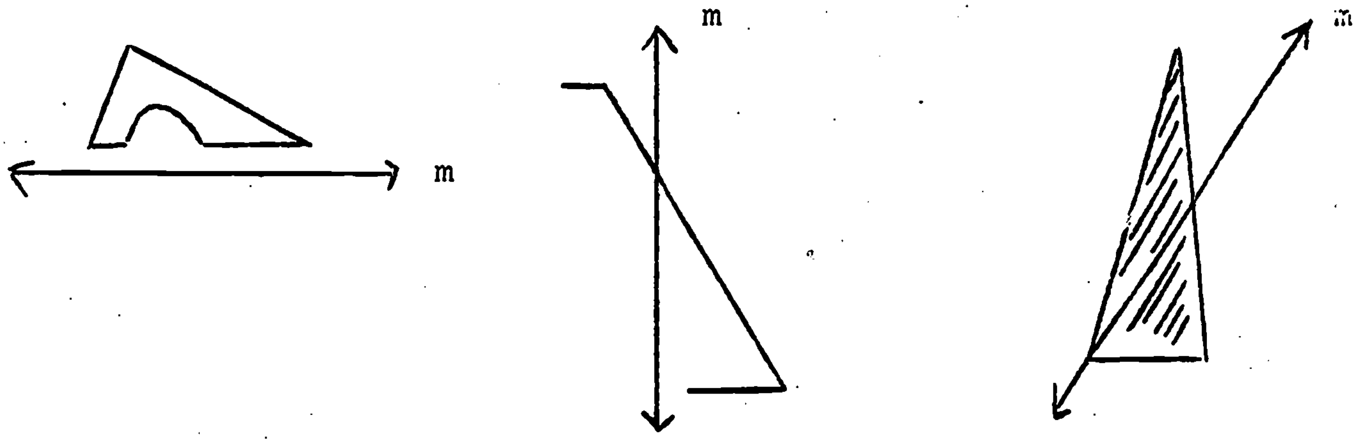
8.) Use mathematical instruments to accurately draw reflections of the following figures. (Draw curved lines freehand).



8.) (cont'd)



i.) Which points did not move ....
These are called .... points.

ii.) Which lines did not change direction after being reflected ....
....

iii.) Which of the following changed after reflection:

a.) lengths of lines Yes No
b.) size of angles Yes No
c.) areas Yes No
d.) shapes Yes No

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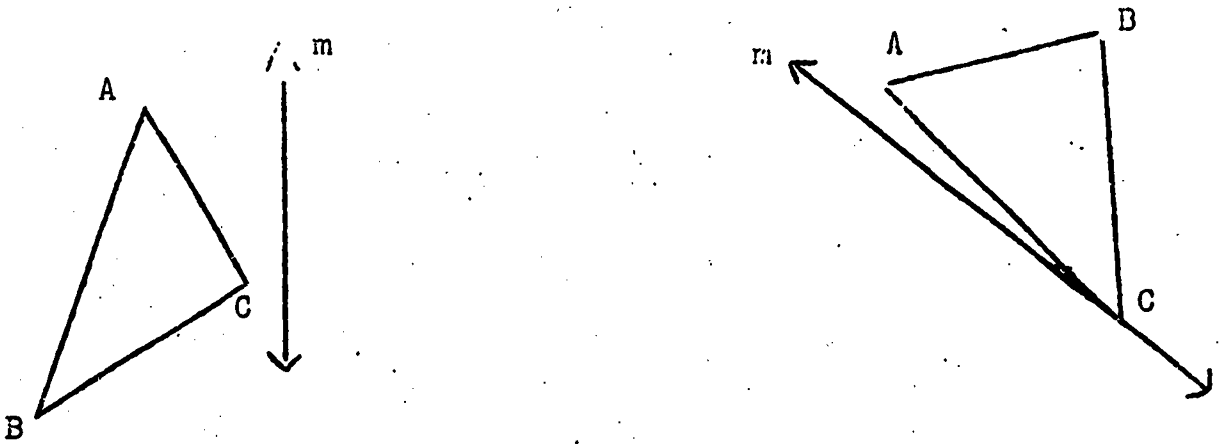
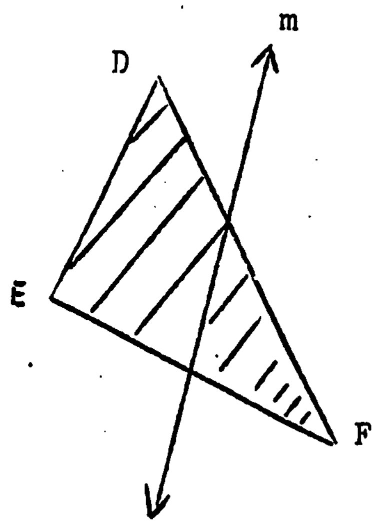
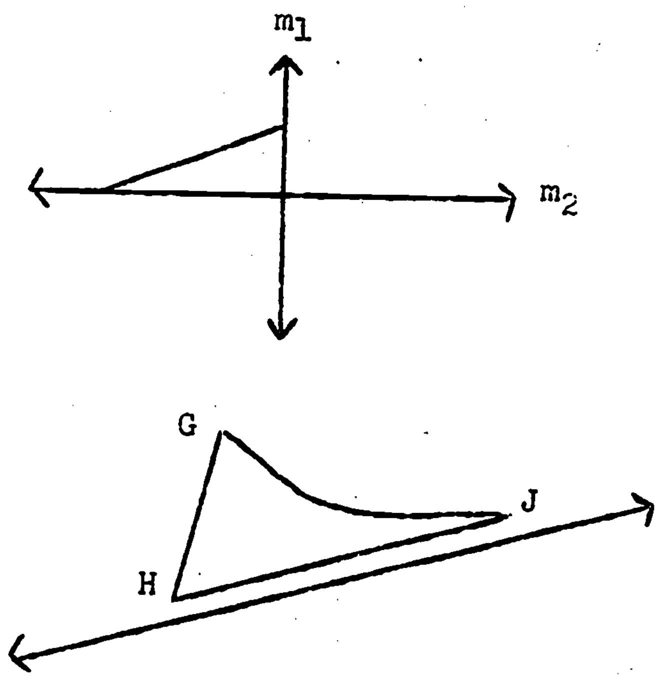
9.) Repeat (8), label image points



9.) (cont'd)





Can you slide figure GHJ onto its image? Yes No

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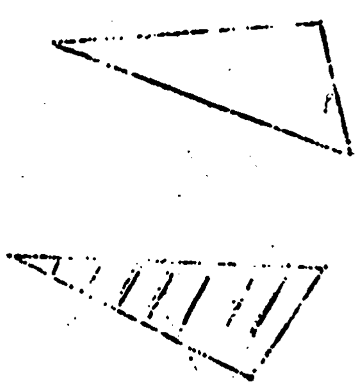
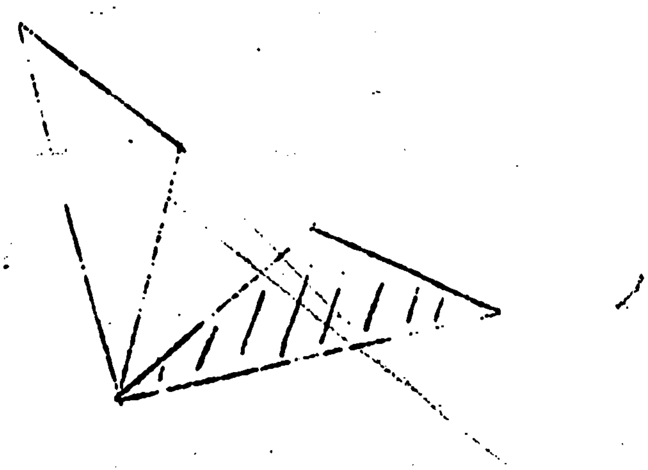
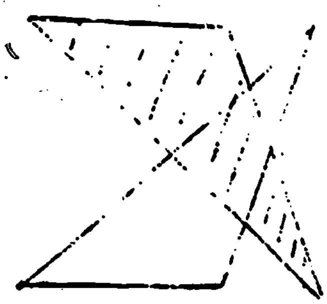
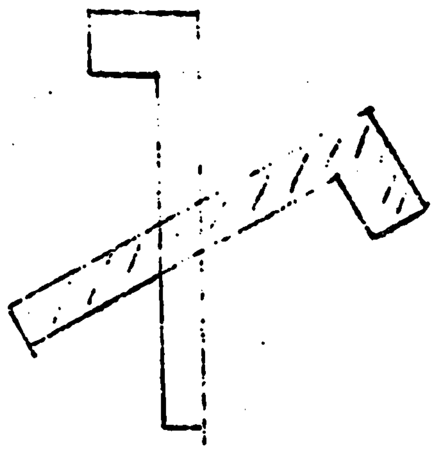
10.) If the shaded shape is the image of the un - shaded shape, use mathematical instruments to accurately show location of "m".





10.) (cont'd)





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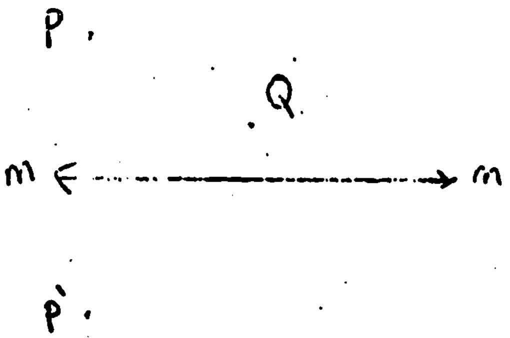
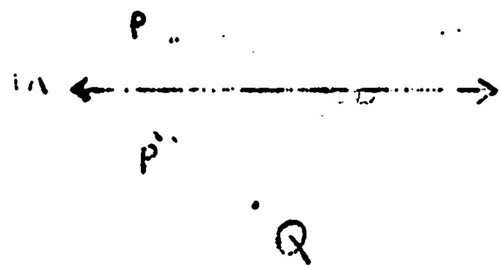
11.) In the following diagrams, accurately locate Q` using only a ruler (and only for the purpose of drawing straight lines).





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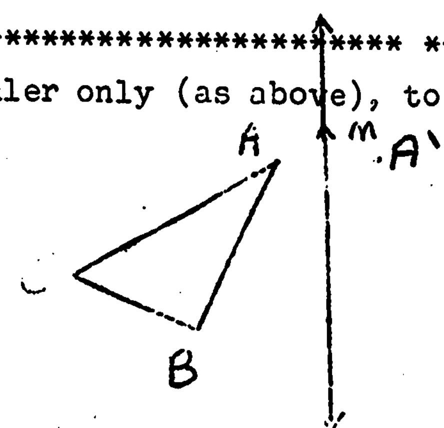
12.) Use a ruler only (as above), to draw △ A` B`C`



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13.) Use a compass only, to locate "m".

1.) A.

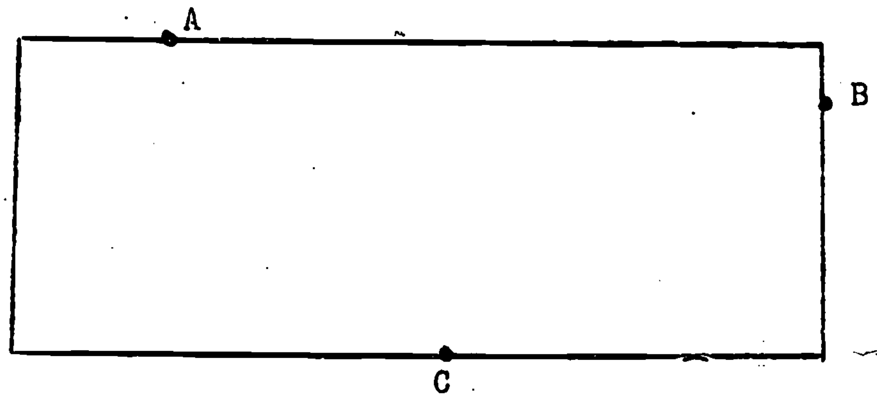
2.) B.

A`.

.B`

************************************ **

14.) Shade in the region on the playing field in which the football is closer to A than to either B or C. (Use accurate construction).



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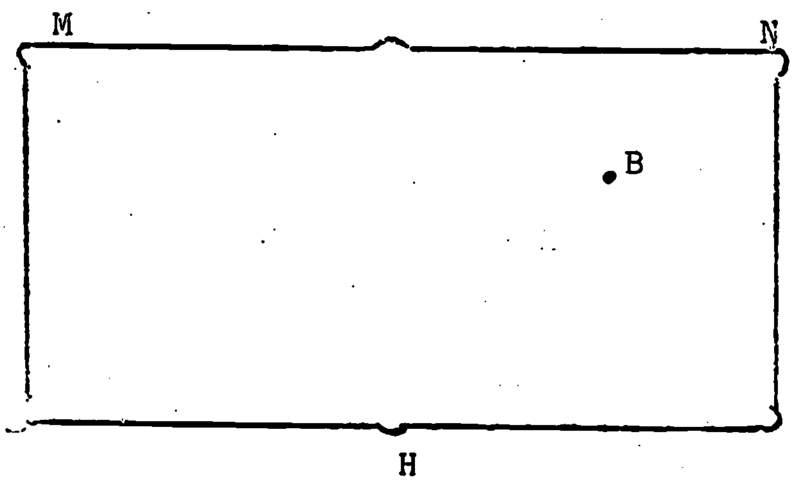
15.)



Draw accurate path of ball B if it is to be hit off edge MN of pool table and roll into hole at H.

What imaginary point could you aim for? ....

....

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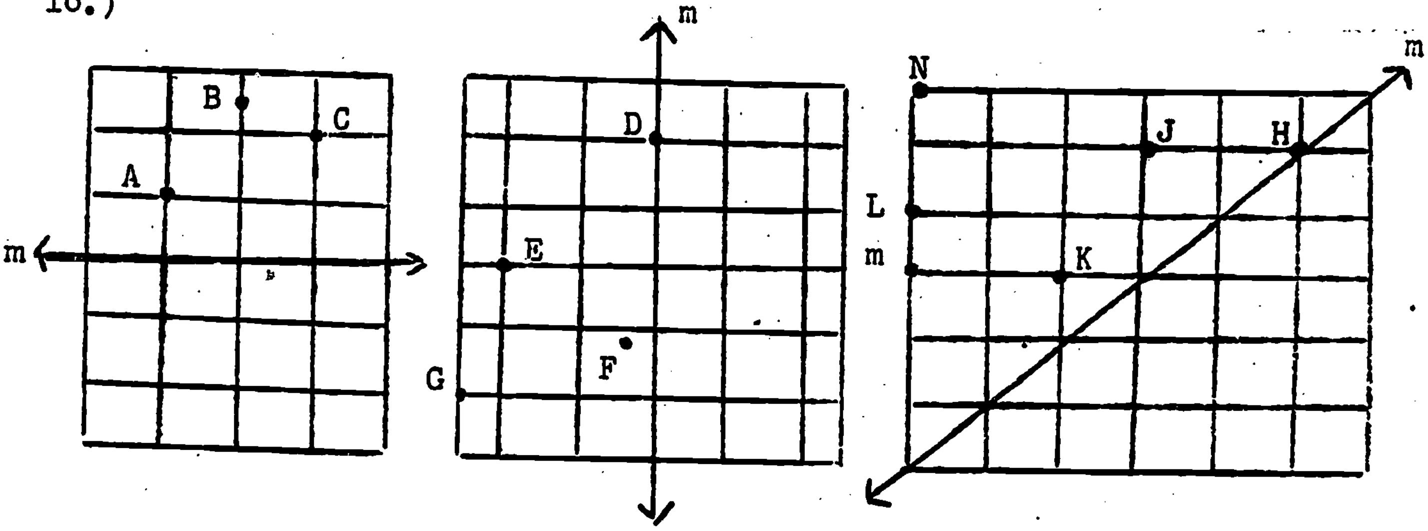
16.)



Show images of above points after reflection in given mirror lines.

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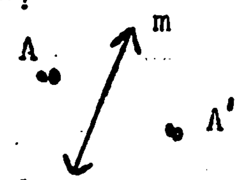
17.) a.) What is the rule defining the movement from A to A' under the operation of reflection in "m"?



....

....

....

b.) Which points remain invariant during a reflection?

....

....

c.) What determines where points will move to?

....

....

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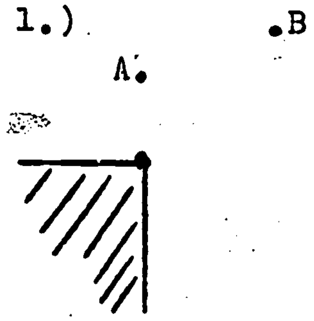
ROTATION:

1.)



On a given signal the two boys A and B are told to move through an angle of 90° about the corner of the school building.

Sketch A' and B'

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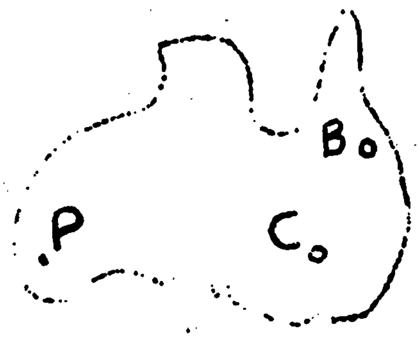
2.) Trace the shape onto cardboard. Cut it out. Keep P invariant, using a compass. Put your pencil through the hole at B and rotate the shape around P in a complete revolution leaving the locus (path) of B.

Repeat for C.



a.) The loci of B and C are ....

b.) .... moved the greater distance

c.) .... moved through the greater angle

Rotation: (cont'd)

2.) Trace the shape. Label B and C.

Find some way of rotating the shape so that it turns 90° clockwise about P.

Label B' and C'

Which point moved through the greater angle? Answer ....

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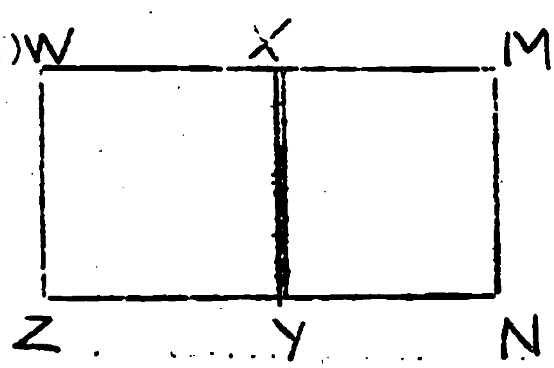
3.)



Figures WXYZ and MNYX are squares.
Describe a rotation that would map the squares onto each other and

a.) Z to X

....

....

b.) M to Y

....

....

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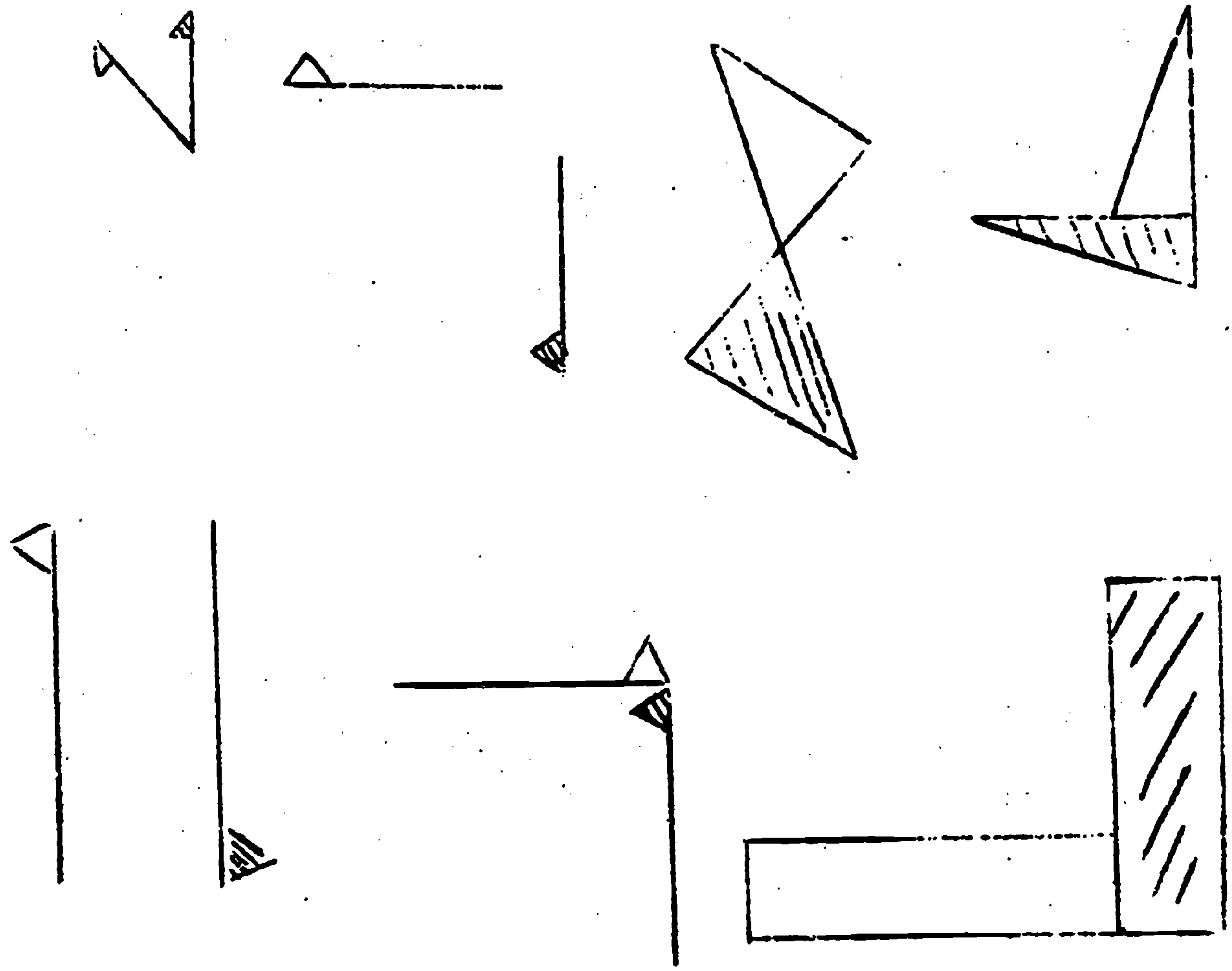
4.) MARK DOTS AT THE CENTRES OF ROTATION:



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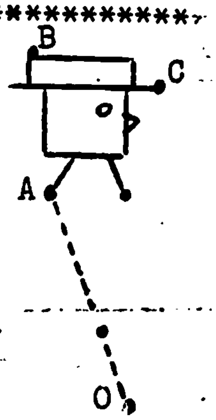
5.)



Draw angle ∠AOX -90° (negative ∴ clockwise from $\overline{OA}$).
Trace the shape and rotate it about O until A is on line OX.
Mark A', B', and C' ....

∠AOA' = ...

Measure ∠BOB' .... Answer ....

Measure ∠COC' .... Answer ....

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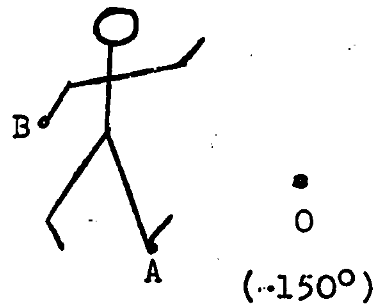
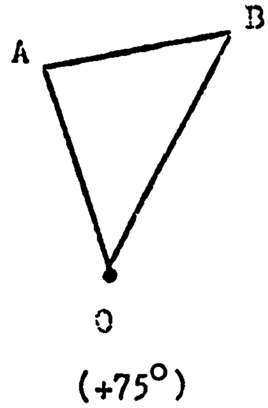
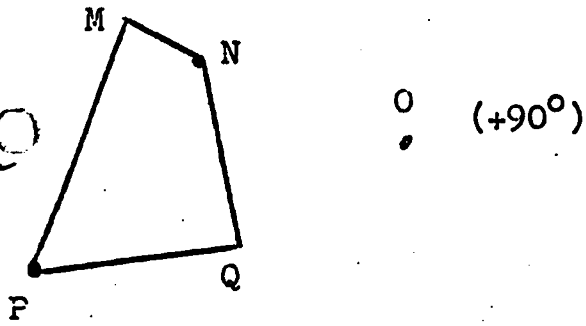
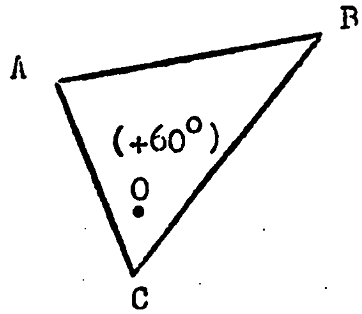
6.) Use a method similar to (5) to rotate these figures the given angles about O.

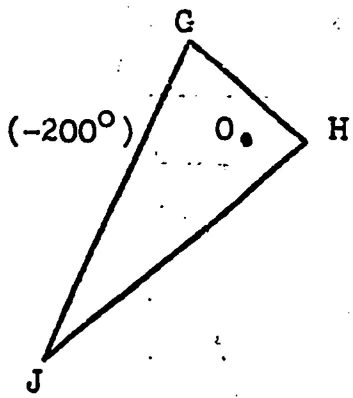
Label image points where possible.

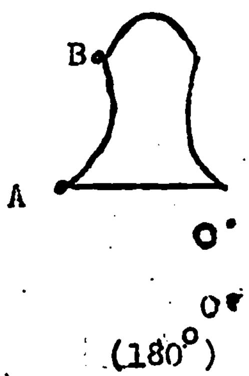












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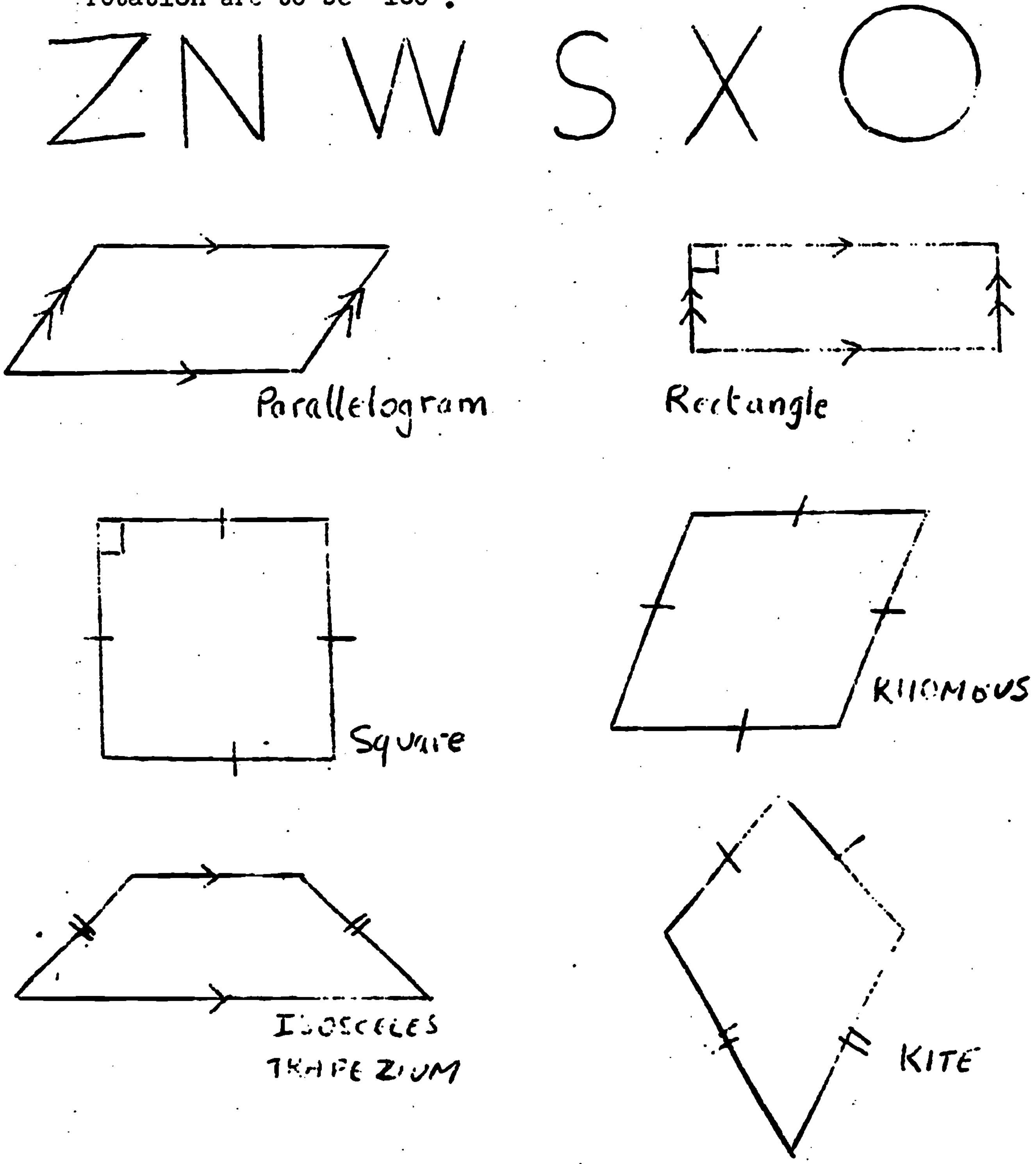
7.) Mark dots at centres of rotation (if possible) if angles of rotation are to be 180°.



CHECK using tracing paper.

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3.)

We assume Δ ABC rotates about some point onto shaded Δ.
Label A`, B`, and C`.
Construct mediators of $\overline{AA`}$, $\overline{BB`}$ and $\overline{CC`}$
What can you say about the mediators ....

....
Show that this point is the centre of the rotation by using tracing paper.
Measure the following angles: a.) $\angle$ AOA` = ...

b.) $\angle$ BOB` = ...

c.) $\angle$ COC` = ...

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9.) Write two sentences describing how you would explain to a new student the method of finding:

a.) the centre of rotation [for a problem like (8)]

....

....

b.) the angle of rotation

....

....

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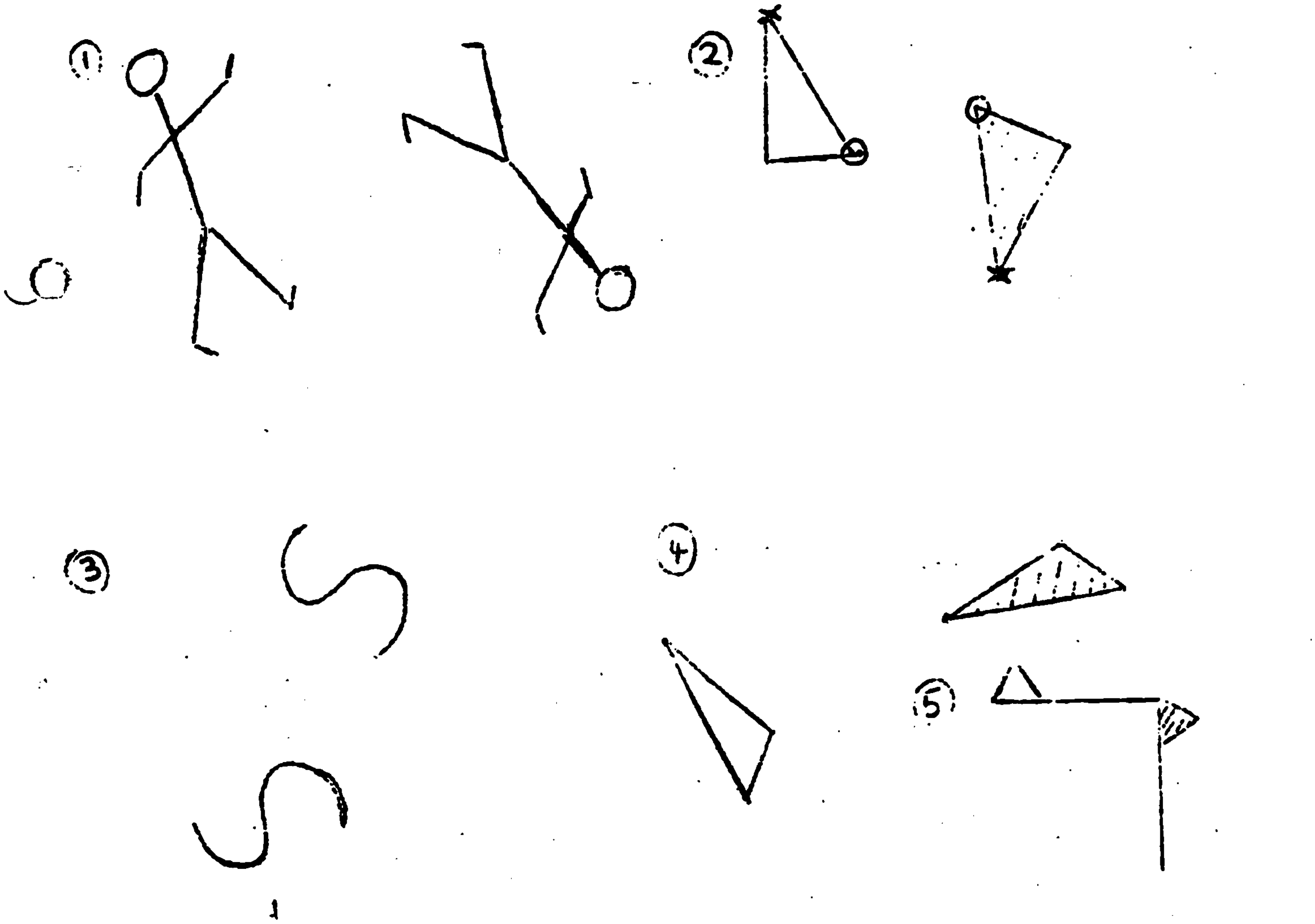
10.) Find the centres and angles of rotation for each of the following transformations.......[check using tracing paper].



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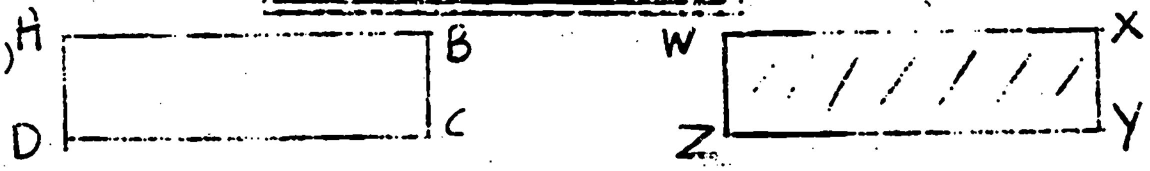
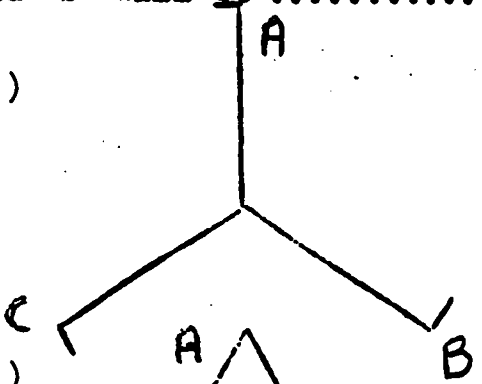
11.) a.)



After rotation of 180° about some point the images of A, B, C, and D will be .... respectively.

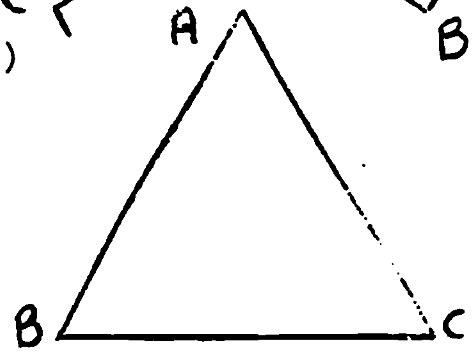
b.)



i.) After a rotation of 120°, A moves to ....

ii.) After a rotation of −120°, A moves to ....

c.)



△ ABC is equilateral

i.) Where would the centre of rotation be?

....

ii.) What rotation could map B to A?

....

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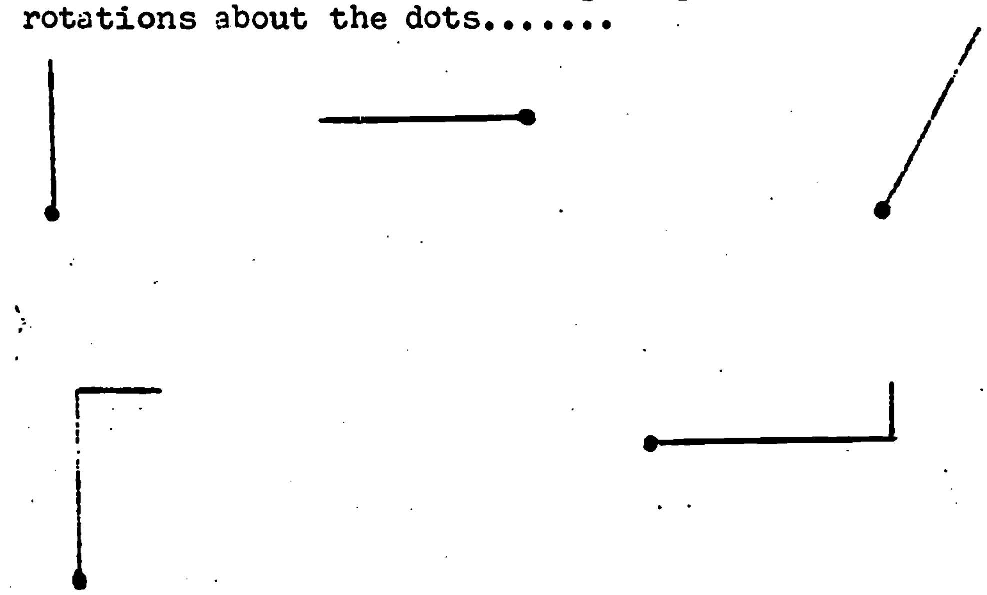
12.) a.) Draw freehand sketches showing images after 90° clockwise rotations about the dots.......



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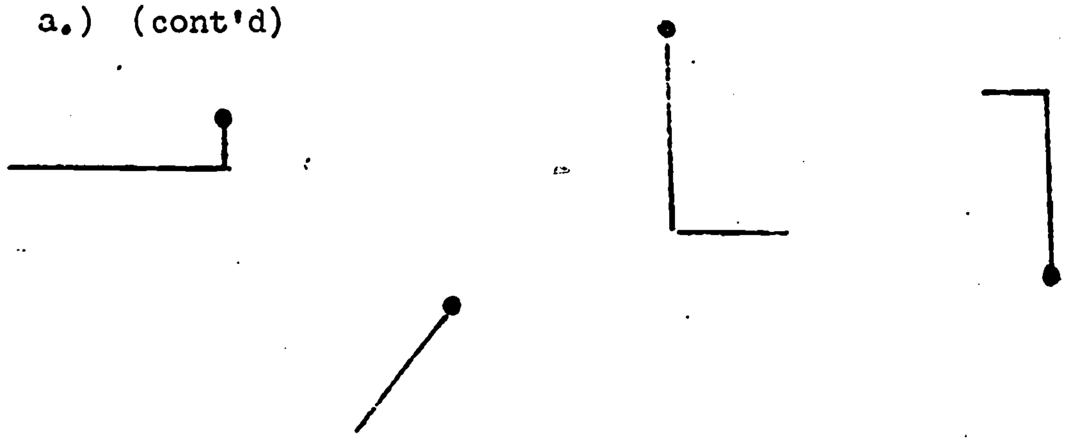
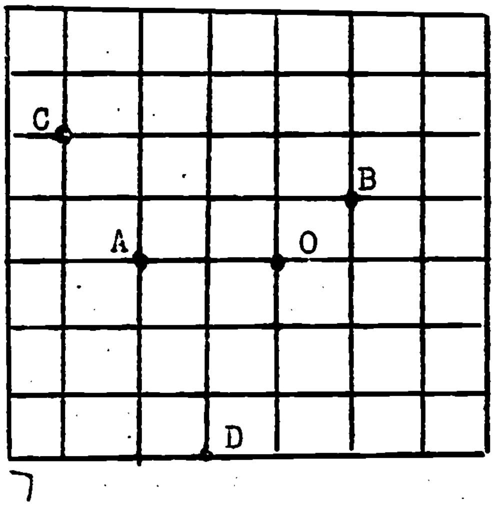
12.) a.) (cont'd)



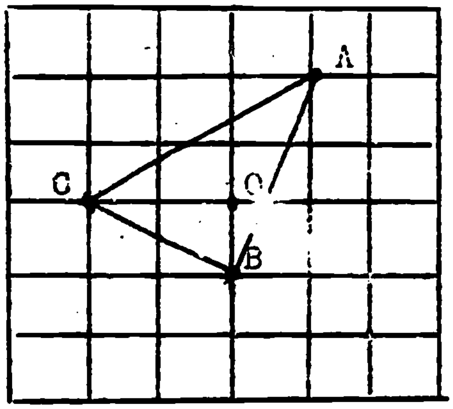
12.) b.)



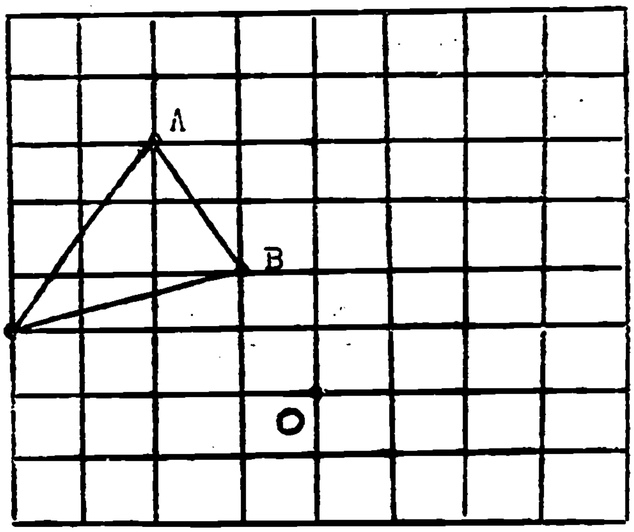
Rotate A, B, C, and D 90° clockwise about O, without using instruments.

12.) c.) Rotate ΔABC, 90° clockwise about O.

(i)



(ii)



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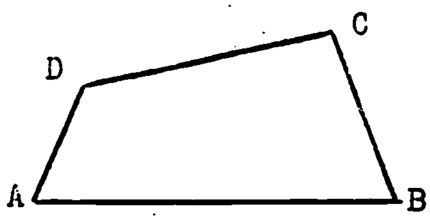
TRANSLATION:

1.) Trace this shape onto cardboard. Cut it out, place your ruler edge along $\overleftrightarrow{AB}$ and slide the figure 3" to the right. Mark A', B'; C', and D'



What can you say about:

a.) AA', BB'; CC' and DD' ....

b.) $\overline{AA'}$; $\overline{BB'}$, $\overline{CC'}$, and $\overline{DD'}$ ....

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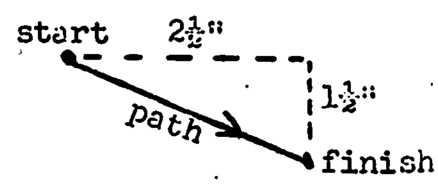
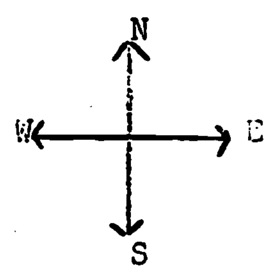
2.) Trace the shape from (1) and translate it according to the rule ($2\frac{1}{2}$" E, $1\frac{1}{2}$" S).

(i.e.)





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3.) Translate this shape (3" E, 1" S)

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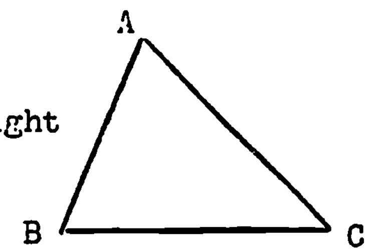
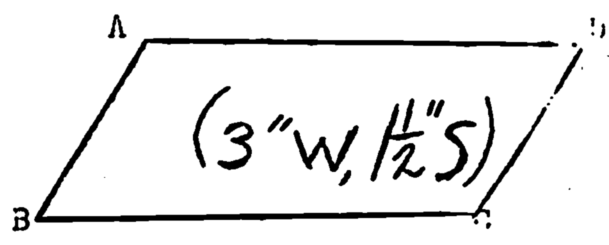
4.) Translate the following figures, using tracing paper

a.) 8cms to right



b.)



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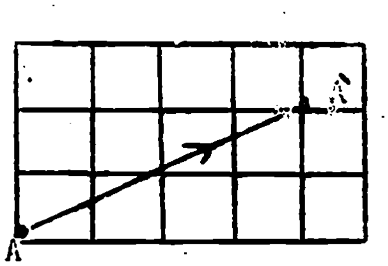
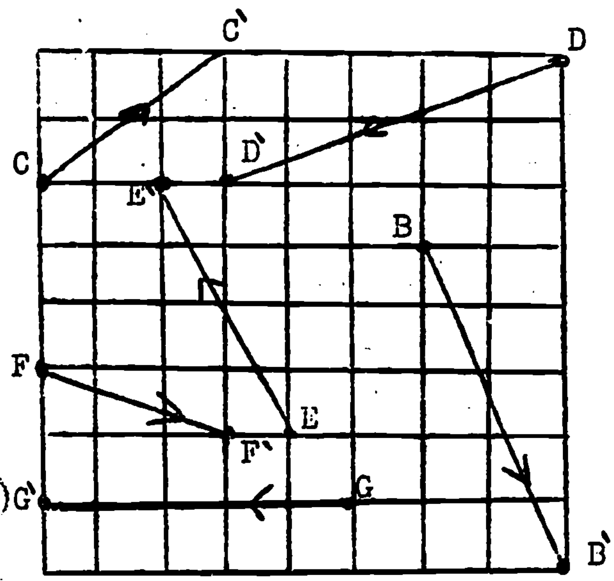
5.)



This translation from A to A' is defined by the rule (+4, +2)

Complete this table:



Translation	Rule
B ⟶ B'	(2, -5)
C ⟶ C'	
D ⟶ D'	
E ⟶ E'	
F ⟶ F'	
G ⟶ G'	

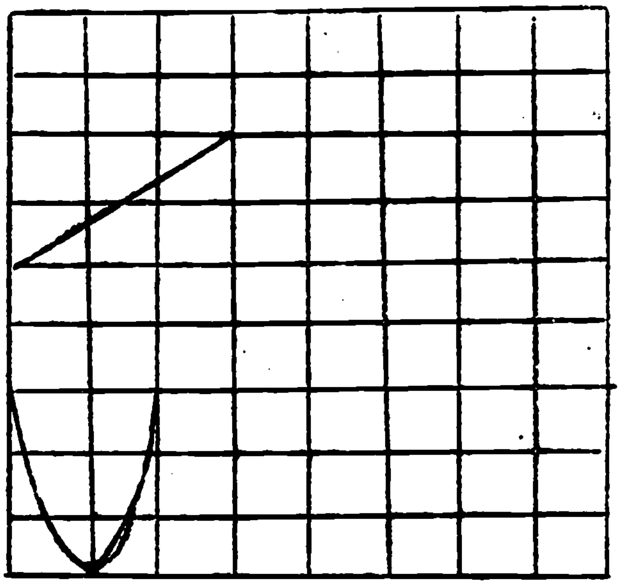
****

6.) Sketch the translations of the following points if the respective rules are:

A(6,1) B(-2,5) C(3,-2) D(-7,-5)

****

7.) Translate the line and the parabola according to the rule: (4, 2)



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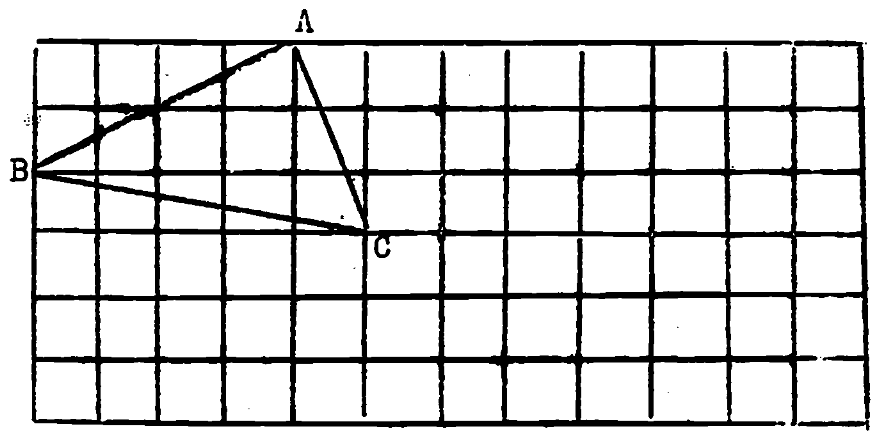
8.) Translate the triangle according to the rule (7, 3)
Label A`, B`, and C`



Tick any of the following which are not changed by a translation.

Sense ☐
Lengths ☐
Angle size ☐
Area ☐
Shape ☐

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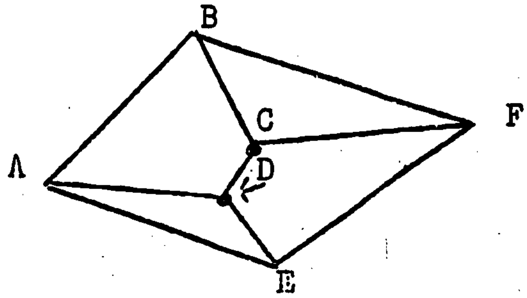
TRANSLATION SUPPLEMENT (OPTIONAL):

1.)



A, B, C, D, E, F are six towns connected by the roads shown.
A journey from A to F via B is written as: $\underline{AB} + \underline{BF}$
This is the sum of two translations.

Write one simple translation equal to the following:

1.) $\underline{AD} + \underline{DC} + \underline{CF}$ = ...
2.) $\underline{DE} + \underline{EA}$ = ...
3.) $\underline{AB} + \underline{BC} + \underline{CF} + \underline{FE}$ = ...
4.) $\underline{AB} + \underline{BC} + \underline{CD} + \underline{DA}$ = ...

How many different ways are there of going from A to F without going through a town twice?

Answer ....

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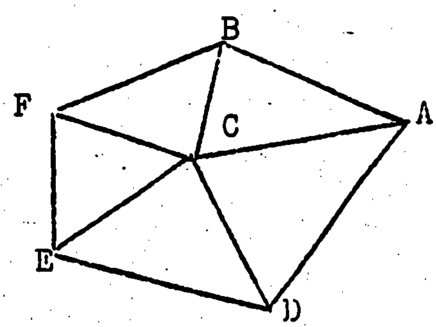
2.)



How many ways are there of going from:

i.) A to C ....
ii.) A to F ....

List the ways of going from A to C:

1.) .... 6.) ....
2.) .... 7.) ....
3.) .... 8.) ....
4.) .... 9.) ....
5.) ....

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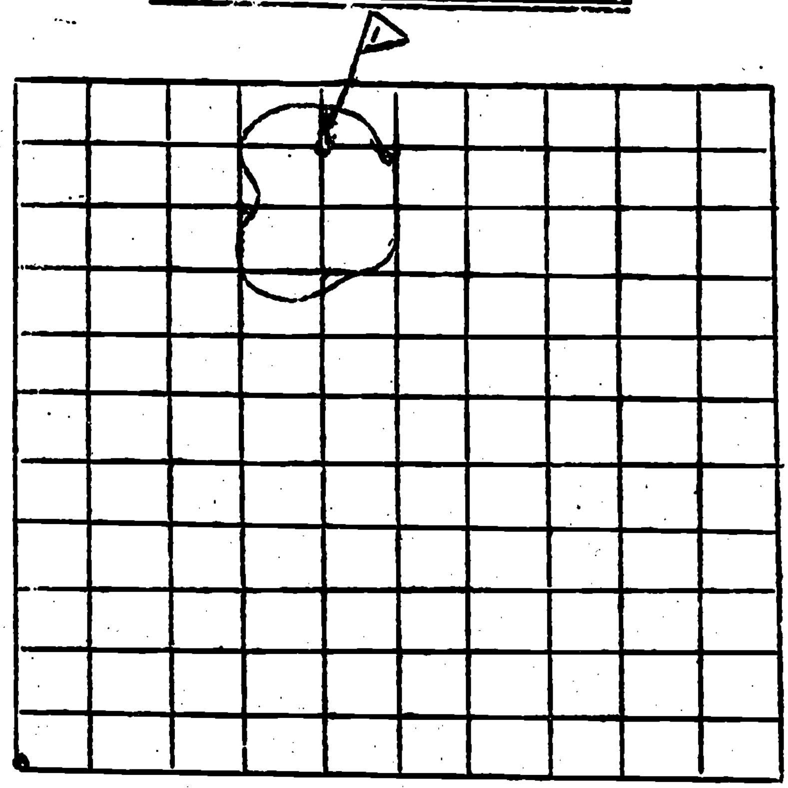
3.)



(0,0)

The hole is at (4, 10)

Sketch a golfer's first shot <u>from</u> (0,0) to (3,6)

Then his second shot of $\overline{(-1,3)}$

What was his final shot if he "holed out"

Answer ....

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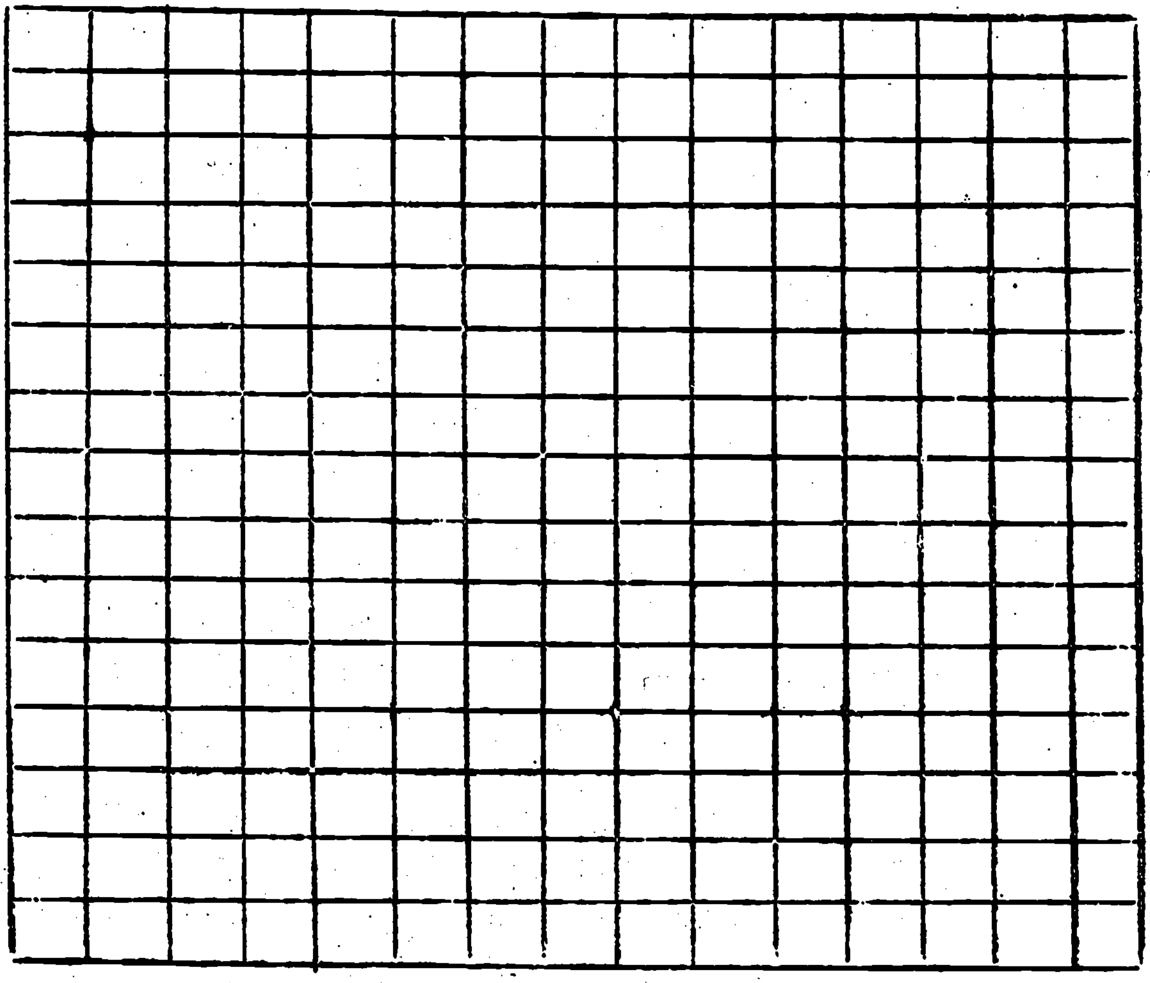
4.) The translation $\overline{(5,2)}$ represents a horizontal displacement of .... and a vertical displacement of ....

Sketch a translation of $\overline{(5,2)}$

********************************* ************************************

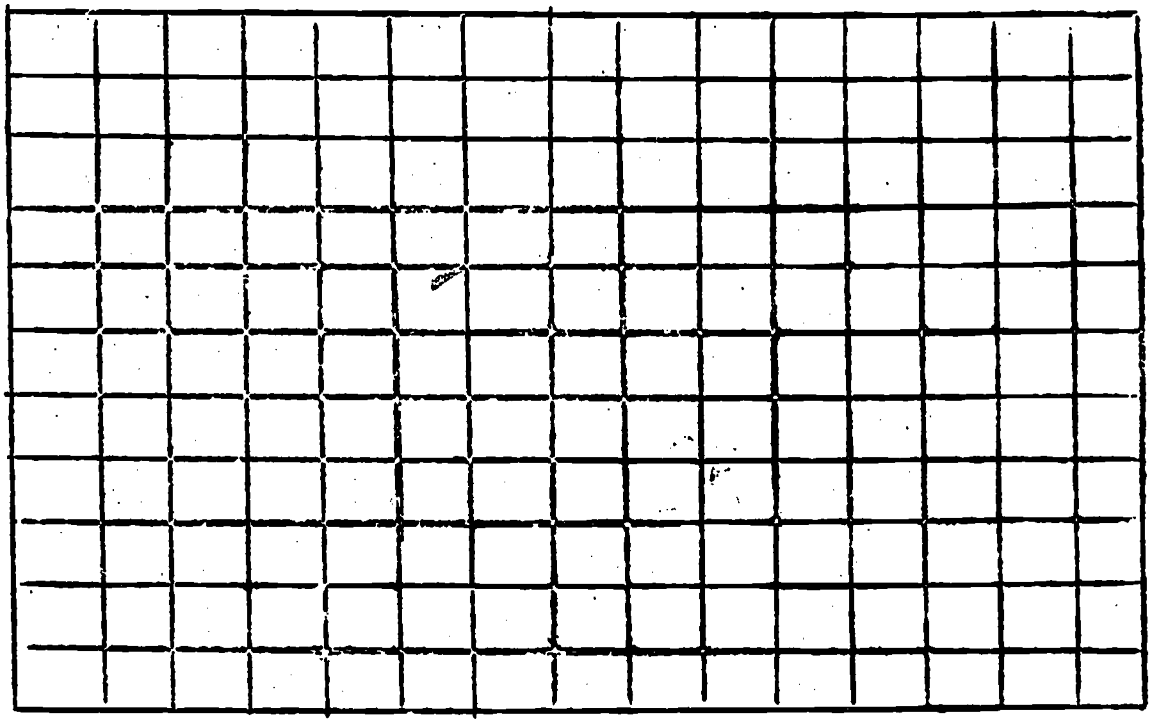
5.)



Use the grid above to assist you in completing this table.

Initial Point	Translation	Terminal Point
(-1,3)	$\overline{(3,-4)}$	
	$\overline{(-2,1)}$	(5,-4)
(3,8)		(3,6)
	$\overline{(-3,-7)}$	(2,-8)
(-2,7)		(-5,9)
(-3,-1)	$\overline{(-1,5)}$	
(2,0)		(0,2)

6.)



Sketch the following translations:

a.) from (0,0) to (3,4)

b.) from (7,0) to (11,3)

c.) from (4,3) to (0,0)

d.) from (5,1) to (1,-2)

e.) from (-3,4) to (1,7)

f.) from (6,3) to (10,5)

Which are quivalent translations?

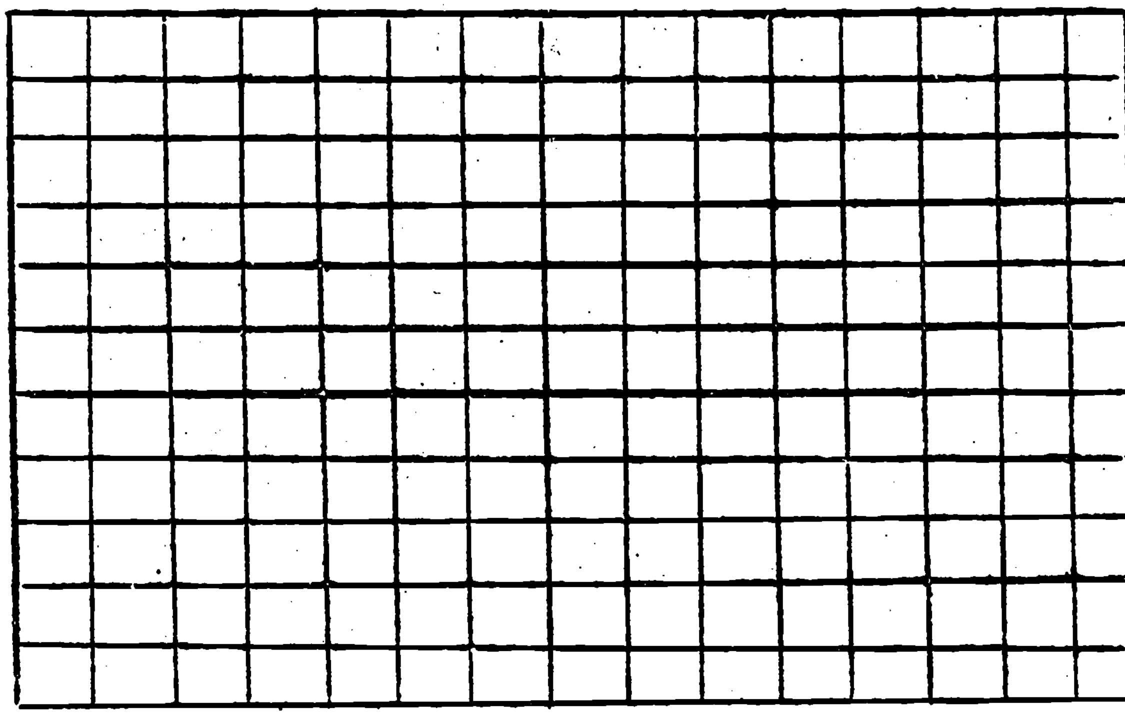
Answers: ....

....

....

....

7.)



1.) IF $\underset{\sim}{a} = \overline{(3,2)}$ and $\underset{\sim}{b} = \overline{(2,5)}$

Sketch $\underset{\sim}{a} + \underset{\sim}{b}$

What single translation is = $\underset{\sim}{a} + \underset{\sim}{b}$

Answer ....

2.) Repeat for $\underset{\sim}{c} = \overline{(-5,1)}$ $\underset{\sim}{d} = (2,8)$

$\underset{\sim}{c} + \underset{\sim}{d} =$ ....

3.) Repeat for $\underset{\sim}{e} = \overline{(3,-1)}$ $\underset{\sim}{f} = \overline{(2,5)}$

$\underset{\sim}{e} + \underset{\sim}{f} =$ ....

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CONGRUENT FIGURES:

Two figures are congruent if one can be mapped onto the other by:

i.) a reflection

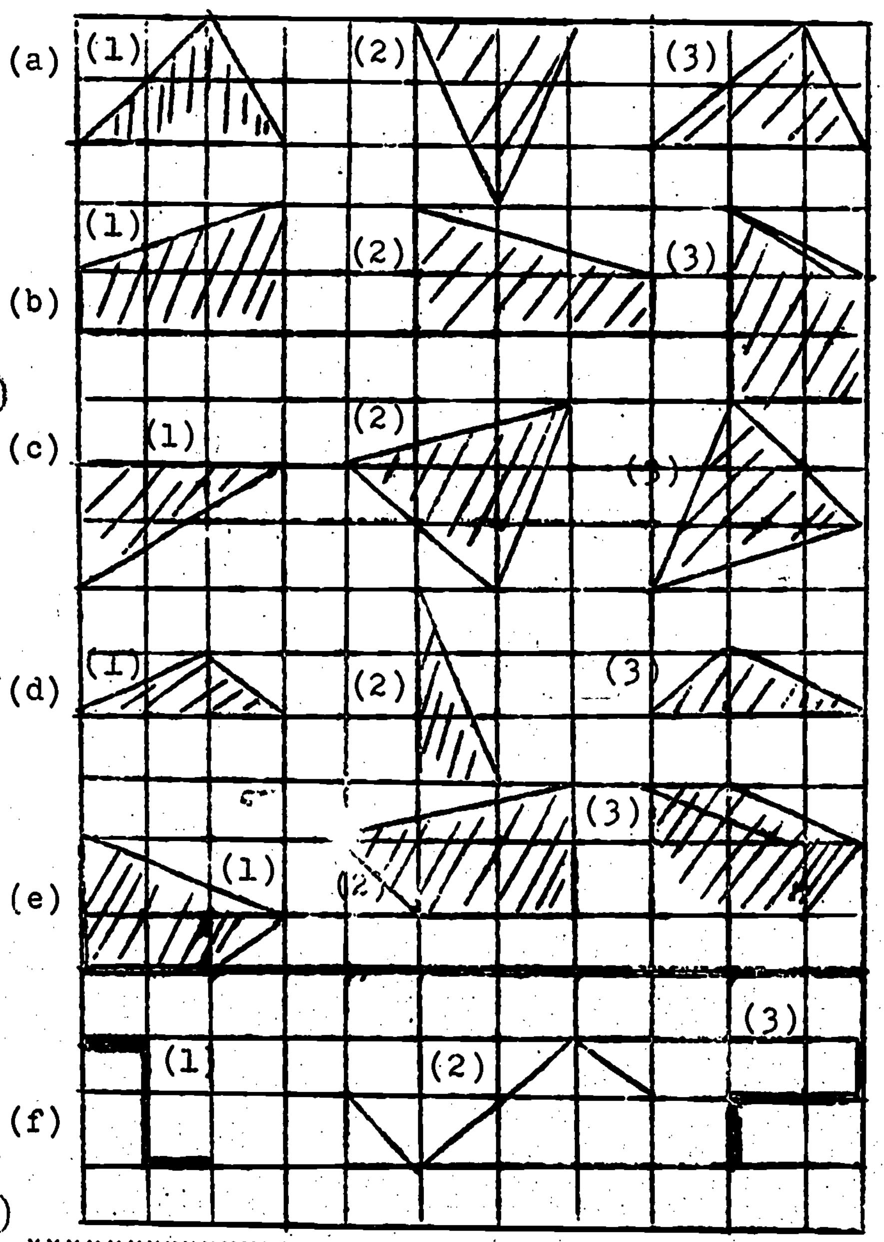
ii.) a rotation

iii.) a translation

or iv.) some combination of (i), (ii), and (iii).

1.) State the pair of figures which are congruent in each case and name the type of transformation.

Answers:



(a)........and.....by.....

(b)........and.....by.....

(c)........and.....by.....

(d)........and.....by.....

(e)........and.....by.....

(f)........and.....by.....

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2.) Complete this table:

Place a tick for each property that doesn't change.

	Sense	Lengths	Angles	Area	Shape
Reflection					
Rotation					
Translation					

Hence, if two figures are congruent they are identical with respect to:

a.) lengths

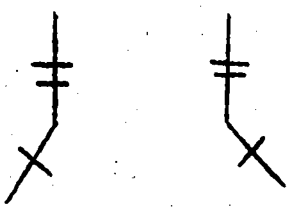
b.) angles

c.) area and shape

****

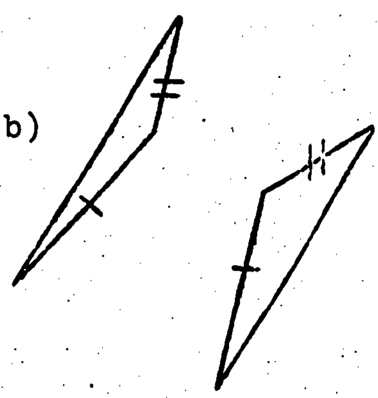
3.) What extra information would you need to know before being certain that the following pairs of figures are congruent (i.e. can be mapped onto each other).

(a)



Answers (a) ....
....
....

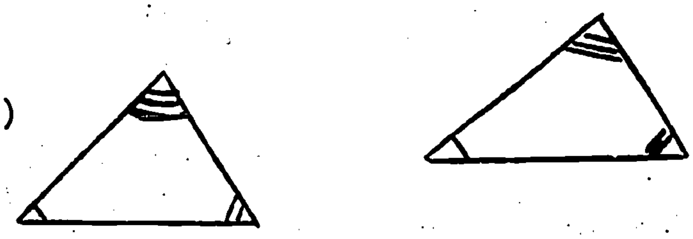
(b)



(b) ....
....
....
....

3.) (cont'd)

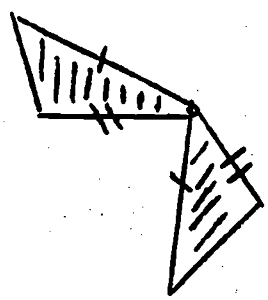
(c)



Answers:

(c) ....
....
....

(d)

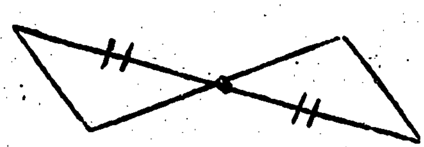


(d) ....
....
....

(e)

(e) ....
....
....

(f)



(f) ....
....
....

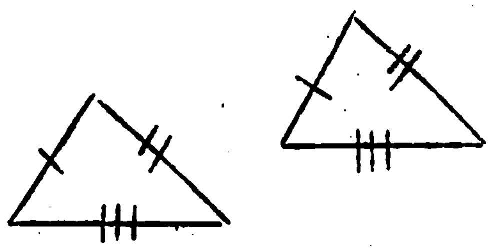
TESTING FOR THE CONGRUENCY OF A PAIR OF TRIANGLES:

We do not need to know that the two triangles are completely identical in lengths, angles, areas and shape.

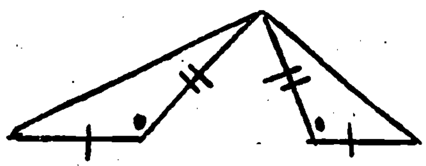
The following are four tests stating minimum conditions which, if satisfied, imply the two triangles are congruent. i.e. can be mapped onto each other.

(1) Are 3 sides identical?
S.S.S.

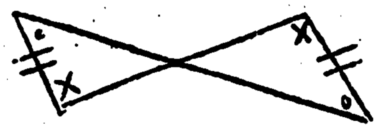
Example:



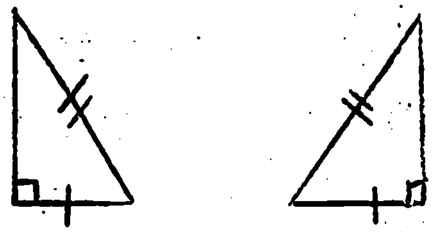
(2) Are 2 sides and the included angle indentical?
S.A.S.



(3) Are 2 angles and one corresponding side identical?
A.S.A.



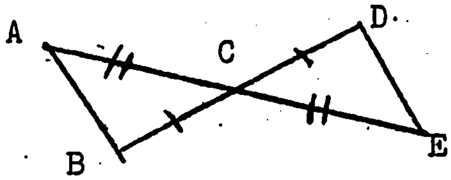
(4) Are the triangles right-angled with equal hypotenuses and another side identical?
R.H.S.



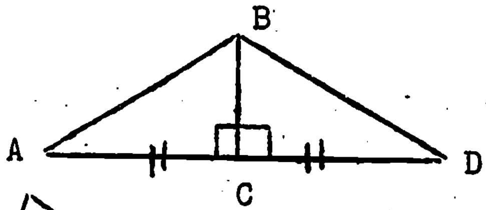
IF ONE OF THE ABOVE TESTS CAN BE PROVED TRUE FOR A PAIR OF TRIANGLES THEN THE TRIANGLES ARE ....

Select pairs of triangles which are congruent. State the transformations connecting them, and briefly state the "certainty" test that gives absolute proof.

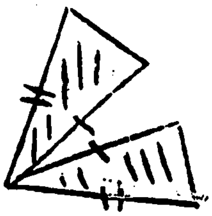
(a)



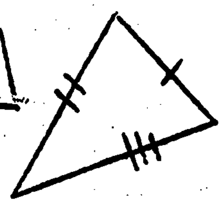
(b)



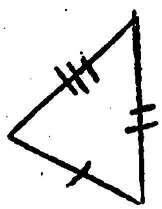
(c)



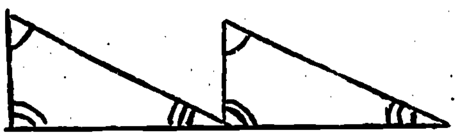
(d)



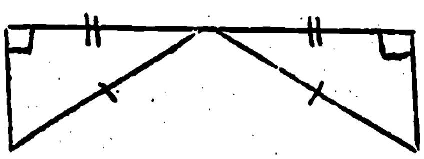
(d)



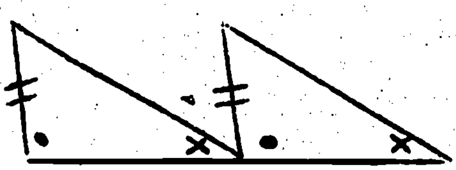
(e)



(f)



(g)



Congruent transformation test

(a) Yes or No ....

Image of ∠A is ....

(b) ....

Image of ∠A is ....

(c) ....

(d) ....

(e) ....

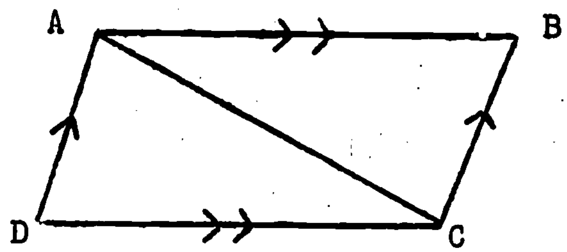
(f) ....

(g) ....

** **

PROPERTIES OF TWO SPECIAL QUADRILATERALS:

1.) Parallelogram



Assume only that opposite sides are parallel.

$\triangle ABC \equiv \triangle$ .... (Test is ....)

The mapping is a ....

1.) $\angle BAC$ = $\angle$ ___

$\angle CAD$ = $\angle$ ___

$\therefore$ $\angle BAD$ =

i.e. ....angles of a parallelogram are.......
....

2.) AB = ... and AD = ...
i.e. .... sides of a parallelogram.............

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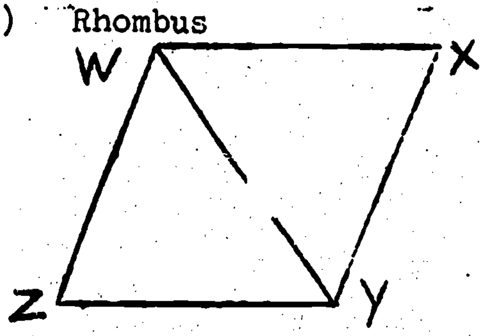
2.) Rhombus



$\triangle WXY \equiv \triangle$..................
under transformation of ....
using .... test

OR

$\triangle WXY \equiv \triangle$..................
under transformation of ....
using .... test

Conclusions $\angle XWY$ = $\angle$ ___ = $\angle$ ___ = $\angle$ ___

i.e. the diagonal of a rhombus....

____

____

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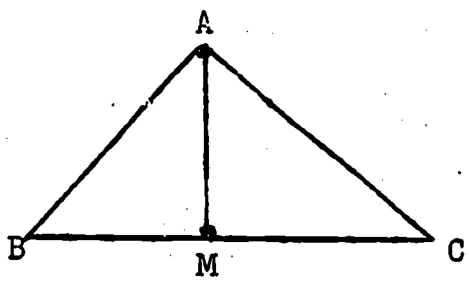
CONGRUENT TRIANGLE PROBLEMS:

1.)



Given △ ABC is Isosceles △.
$\overline{AM}$ is drawn from A to midpoint of base $\overline{BC}$

Prove (with reasons)
AM is at right angles to the base.

Proof (step by step, please)

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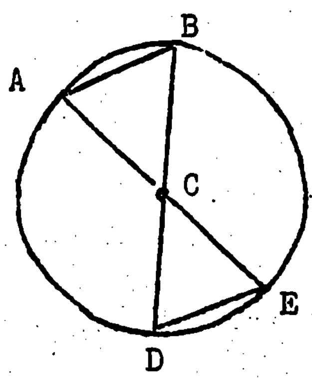
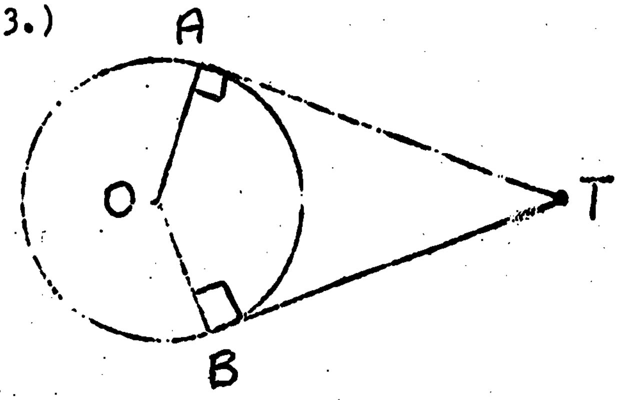
2.)



Given O is centre of circle
Prove AB = DE
Proof

************************************ ************************************

3.)



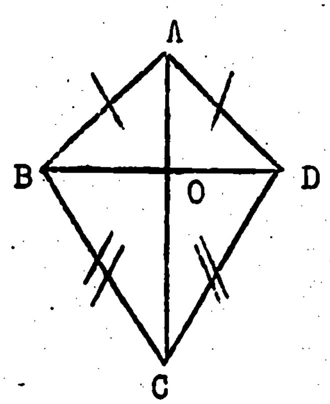
Given (1) O is centre
(2) $\angle A = \angle B = 90^\circ$

Prove AT = BT

Proof

************************************ **

4.)



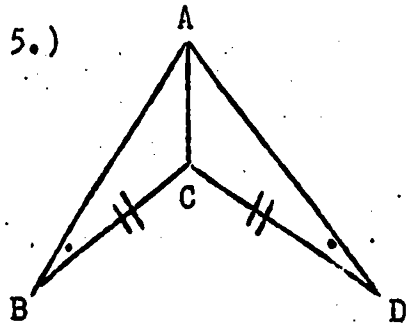
Given Kite ABCD as shown

Prove (1) $\triangle ABC \cong \triangle ADC$
(2) $\triangle AOB \cong \triangle AOD$

Proof

************************************ **

5.)



Given Figure as shown

Prove AB = AD

Proof

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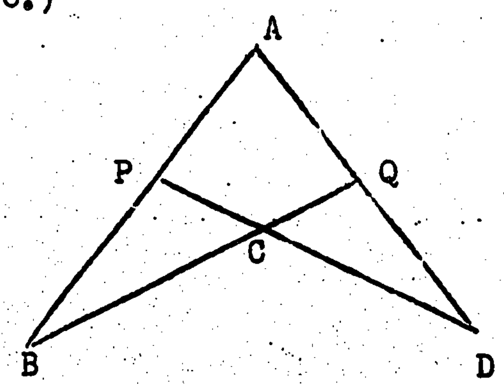
6.)



Given (1) PD = QB

(2) BP = DQ

Prove AP = AQ

Hint Prove △ ABD is isosceles by proving its base angles are equal

Proof

(space for proof provided on next page, for question #6)

6.) (cont'd)

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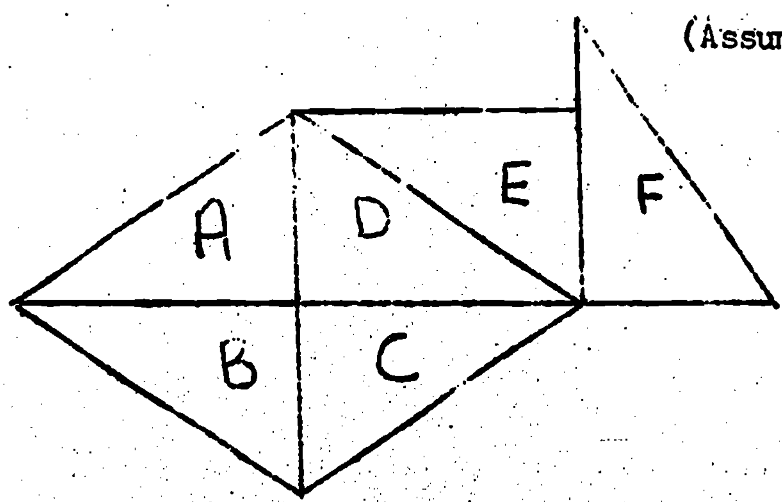
7.) State the type of transformation (or combination) that maps:

1.) A onto B ....
2.) B onto C ....
3.) D onto B ....
4.) B onto E ....
5.) B onto F ....
6.) A onto F ....
7.) F onto E ....

(Assume all △ 's are congruent)



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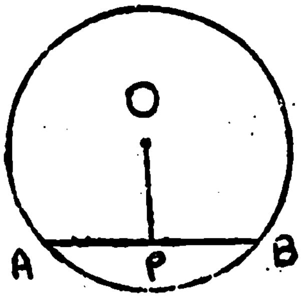
8.)



Given (1) O is centre
(2) $\overline{OP}$ is perpendicular to $\overline{AB}$

Prove P's midpoint of $\overline{AB}$

Proof

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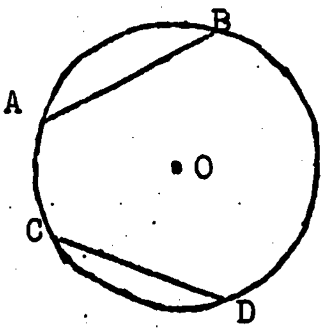
9.)



Given AB and CD are equal chords

Prove $\overline{AB}$ and $\overline{CD}$ are equidistant from the centre O

Proof

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